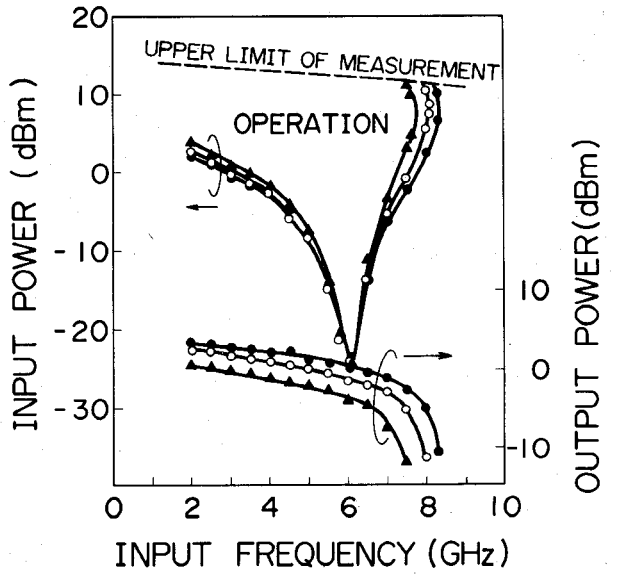

Fig. 4. Frequency characteristics at different supply voltages. Triangles mark data recorded for operation at −4.5 V, open circle at −5.0 V, and closed circle at −5.5 V.

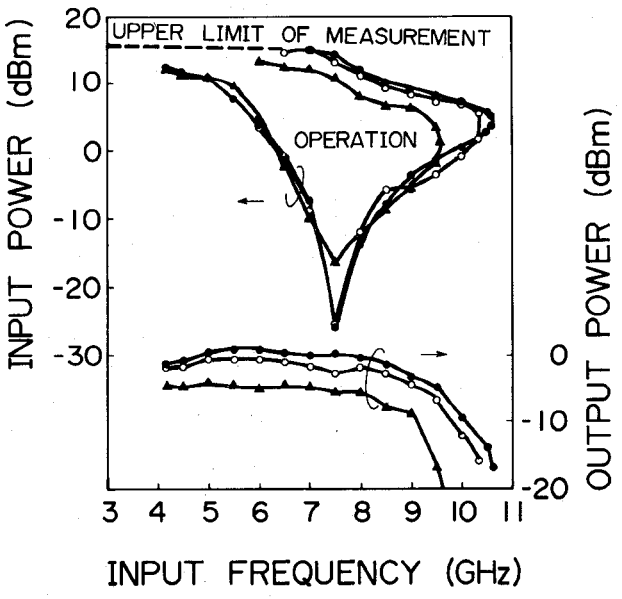

Fig. 5. Frequency characteristics at different supply voltages. Triangles mark data recorded for operation at −6.3 V, open circle at −7.0 V, and closed circle at −7.7 V.

IV. Summary

New 2.0–8.0 GHz and 6.0–10.5 GHz dynamic frequency dividers have been developed. The divider was constructed with a double-loop connected pair of differential amplifiers. The frequency divider was operated from one voltage supply at −5 V, or −7 V with ±10 percent voltage supply fluctuation. This divider overcomes the weak points of conventional dynamic dividers, which need two voltage supplies and precise supply voltage control. This divider operates at a 50 percent higher frequency than static dividers.

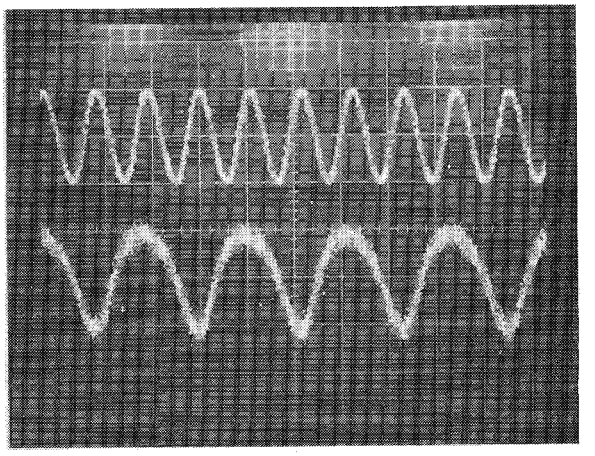

Fig. 6. Input and output waveforms of the frequency divider. Top: input waveform. Bottom: output waveform. 20 dB attenuator inserted between D.U.T. and sampling oscillator. Vertical axis: 50 mV/div. (input); 10 mV/div. (output). Horizontal axis: 100 ps/div.

Acknowledgment

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Properties of Shielded Cylindrical Quasi-TE_{0nm}-Mode Dielectric Resonators

JERZY KRUPKA

***Abstract*—Comparison of the Rayleigh–Ritz method and the mode-matching method for computations of quasi-TE_{0nm}-mode frequencies and unloaded Q factors of shielded dielectric resonators is presented. Rigorous bounds for the true quasi-TE_{0nm}-mode frequencies are assessed. Influence of various parameters on the resonant frequencies, unloaded Q factors, and the temperature coefficients of the resonant frequency is demonstrated for many shielded dielectric resonator structures. Different approaches to unloaded Q factor computations are discussed and numerically compared.**

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I. Introduction

During the last two decades many rigorous methods of analyzing dielectric resonators (DR's) have been developed, such as the mode-matching method [1]–[4], the finite element method [5], [6], the finite difference method [7], and the Rayleigh–Ritz method [8], [9]. However, studies comparing various methods have not appeared in the literature yet.

The purpose of this paper is twofold. The first is to compare the results of computations of the resonant frequencies and unloaded Q factors Q_u of quasi-TE_{0nm}-mode DR's using the mode-matching and Rayleigh–Ritz techniques. The second is to present numerical results illustrating the influence of various parameters on the resonant frequencies, Q_u factors, and the temperature coefficients for many shielded quasi-TE_{0nm}-mode DR's. The mode-matching and Rayleigh–Ritz methods have been chosen for analysis, since they provide lower and upper bounds for the true resonant frequencies of lossless quasi-TE_{0nm}-mode resonant systems. Therefore in such cases the accuracy of computations can be rigorously assessed.

II. Theory

A. Methods of Unloaded Q-Factor Computations

Applications of the mode-matching method and the Rayleigh–Ritz method for computation of the resonant frequencies and unloaded Q_u factors of cylindrical quasi-TE_{0nm}-mode DR's have been described in the literature, e.g. [1]–[4], [8], [9]. Since for Q_u-factor computations we have certain alternative approaches, we consider this problem in detail. The resonant system we are concerned with is sketched in Fig. 1. The most general approach of Q_u-factor computation is a conception of the complex frequency which is introduced in Maxwell's equations and then in the characteristic equation. This method makes is possible, in the source-free region of the resonant system, to determine simultaneously the resonant frequencies ($f = \mathrm{Re}(\omega)/2\pi$) and Q_u factors ($Q_u = \mathrm{Re}(\omega)/2\mathrm{Im}(\omega)$) of the system. All losses can be simultaneously considered. Dielectric losses and conductor losses can be represented in the same manner as imaginary parts of permittivities [4]. Since this method is complicated for low-loss systems (nonradiating systems whose frequencies do not depend significantly on losses), other methods of Q_u-factor computation are commonly used. Lossless systems containing only perfect conductors and dielectrics are considered as the first step in these methods, and the resonant frequencies and the field distributions of the systems are found. Then, as the second step, the Q_u factor is computed using the definition: $Q_u = \omega$ (the energy stored)/(the average power dissipated), assuming that for lossy resonant systems the electromagnetic field distributions remain the same as for the lossless systems. Then we can express the reciprocal of the Q_u factor as follows [10]:

$$Q_u^{-1} = Q_d^{-1} + Q_c^{-1} \tag{1}$$

$$Q_d^{-1} = \sum_{i=1}^{I} p_{ei} \tan\delta_i \tag{2}$$

$$Q_c = A/R_s \tag{3}$$

$$p_{ei} = W_{ei}/W_e = \left(\int_{V_i} \epsilon_i |\vec{E}_i|^2\, dv\right) \Big/ \left(\sum_{i=1}^{I} \int_{V_i} \epsilon_i |\vec{E}_i|^2\, dv\right) \tag{4}$$

$$A = \left(\omega \sum_{i=1}^{I} \int_{V_i} \epsilon_i |\vec{E}_i|^2\, dv\right) \Big/ \left(\int_S |\vec{H}_t|^2\, ds\right) \quad (\Omega) \tag{5}$$

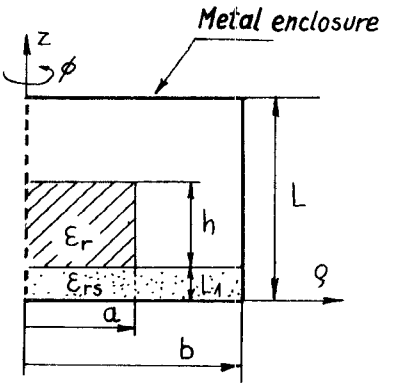

Fig. 1. Cylindrical dielectric resonator on a substrate in a metal shield.

where

- I number of subregions (V_i) of the resonant structure having different loss tangent ($\tan\delta_i$) values,
- S surface of the metal shield,
- R_s surface resistance of the metal shield,
- $\vec{E}, \vec{H}$ electric and magnetic fields of the lossless resonant system.

The coefficients p_{ei} (called the electric energy filling factors) and A, which appear in the above equations, depend only on permittivities and on relative dimensions of the system, so the results for p_{ei} and A are representative for resonators operating at any frequency value. There are two methods for evaluation of the integral ratios appearing in (4) and (5). In the first the electromagnetic fields are substituted into (4) and (5) and integrations are performed directly. In the second, perturbation theory is used. Cavity material perturbation theory is used for p_{ei} computations [3] and cavity wall perturbation theory (incremental frequency rule) [11] for A-factor computations. In the last case, the theory is valid only for resonant systems whose normal components of the electric field vanish on the metal shield. We can also mix different formalisms. For example, depending on dielectric losses, the Q_d factor can be computed using complex frequency formalism, while the Q_c factor, depending on conductor losses, can be computed by means of perturbation theory.

B. Methods of Temperature Coefficient Computation

Due to temperature variations, each material expands with its own temperature expansion coefficient and each dielectric constant changes its value with its own temperature coefficient of dielectric constant. When we evaluate all the differentials for the resonant system shown in Fig. 1, we obtain the following formula for the relative change of frequency due to temperature [10]:

$$\tau_f = C_L\alpha_L + C_b\alpha_b + C_h\alpha_h + C_{L1}\alpha_{L1} + C_{\epsilon r}\tau_{\epsilon r} + C_{\epsilon rs}\tau_{\epsilon rs} \tag{6}$$

where the α's are the temperature expansion coefficients and the τ's are the temperature coefficients of frequency or dielectric constants. The C coefficients can be evaluated numerically as

$$C_x = \frac{\partial f}{f} \Big/ \frac{\partial x}{x} = \frac{\partial f}{\partial x} \cdot \frac{x}{f} \tag{7}$$

where x denotes any parameter (dimension or permittivity).

Usually the metal shield is made of homogeneous metal and the dielectric of the DR is isotropic so $\alpha_L = \alpha_b = \alpha_m$ and $\alpha_a =$

$= \alpha_d$. Then we can rewrite (6) as follows:

$$\tau_f = C_m\alpha_m + C_d\alpha_d + C_{L1}\alpha_{L1} + C_{\epsilon r}\tau_{\epsilon r} + C_{\epsilon rs}\tau_{\epsilon rs} \tag{8}$$

where

$$C_d = C_a + C_h$$
$$C_m = C_b + C_L.$$

It is well known that if all dimensions of a resonant system are increased by a certain factor, the resonant frequency of the system decreases by the same factor. As a result, the coefficients C_m, C_d, and C_{L1} satisfy the following equation:

$$C_m + C_d + C_{L1} = -1. \tag{9}$$

If two coefficients are known (one if $L_1 = 0$), the remaining one can be calculated from (9). The coefficients $C_{\epsilon r}$ and $C_{\epsilon rs}$ are related to the electric energy filling factors. It can be proved from the material perturbation theorem that [3]

$$|C_{\epsilon i}| = (2W_e/W_{ei})^{-1} = p_{ei}/2. \tag{10}$$

The coefficients C_L, C_b, and C_{L1} are related to the A factor. We can prove from the cavity wall perturbation theorem that

$$A = \omega\mu_0 L/[2(|C_L| + |C_{L1}|L/L_1 + |C_b|L/b)] \tag{11}$$

(for $L_1 = 0$, $|C_h|L/h$ instead of $|C_{L1}|L/L_1$ must be substituted in (11)).

III. Results of Computations

Most numerical results are presented in general form independent of absolute dimensions, physical constants, and loss parameters. Since for cylindrical rod DR's, the TE_{011}-mode frequency, denoted in this paper by f_{rod}, is usually known, we have reduced the $TE_{01\delta}$ (quasi-TE_{011}) frequencies with respect to f_{rod} values. Higher order quasi-TE_{0nm}-mode frequencies have been reduced with respect to the velocity of light. For cavity-type resonant systems the classical Rayleigh–Ritz method (R) has been used with empty cavity TE_{0nm}-mode basis (100 basis functions), while for open-type systems the modified Rayleigh–Ritz method (MR) [9] has been used with rod resonator TE_{0nm}-mode basis (25 basis functions). The radial mode-matching method (M-M) has been used with nine functions in each of the two complementary regions. For these numbers of basis functions all methods require approximately the same time for computation (approximately 10 seconds on a CDC 6600 computer per frequency value). The computer program for the modified Rayleigh–Ritz method has been written in complex version, while two others are in real versions. Some of the results have been obtained by a complex version of the mode-matching method (the computer program has been written by Sz. Maj).

We denote different methods of A and p_{ei} factor computations as follows: C for the complex frequency method, I for the incremental frequency (perturbation) method, and D for the method using direct integration of the electromagnetic fields.

Table I gives results of computations for a cavity-type resonator having $\epsilon_r = 10$, $a/h = 1$, $b/L = 1$, and $L_1 = 0$. It is seen that the frequency values computed by the Rayleigh–Ritz method and by the mode-matching method agree to within 0.15 percent for any $p = h/L$ value. The maximum discrepancy appears for $p = 0.25$. Also, A-factor values agree to within 0.35 percent if the incremental frequency rule is used in both methods. If the direct integration method (D) is used, differences in A-factor values obtained by the mode-matching and Rayleigh–Ritz methods are somewhat greater. For interior modes (modes whose energy is predominantly concentrated in the dielectric) the electric energy filling factor values, computed by both methods, agree to within 0.1 percent. Greater discrepancy occurs for exterior modes.

TABLE I
The Computed Values of Normalized $TE_{01\delta}$-Mode Frequencies f/f_{rod}, the Electric Energy Ratio p_e^{-1}, A, and C_m Coefficients versus h/L for a DR Situated in a Cylindrical Cavity

h/L	f/f_rod		p_e^-1			A			C_m
	M-M	R	M-M(I)	R(I)	R(D)	M-M(I)	R(I)	R(D)	M-M
0.00	0.0000	0.0000	∞	∞	∞	665.7	665.7	665.7	-1.000
0.15	0.5390	0.5390	364.37	380.20	380.80	660.8	661.2	660.8	-0.994
0.20	0.7116	0.7118	14.12	14.76	14.80	623.3	624.4	624.5	-0.885
0.25	0.8030	0.8042	1.443	1.442	1.443	519.1	519.3	529.8	-0.181
0.30	0.8181	0.8188	1.171	1.171	1.171	514.7	516.5	501.8	-0.065
0.35	0.8253	0.8257	1.118	1.118	1.118	516.5	517.0	516.8	-0.054
0.40	0.8312	0.8315	1.092	1.092	1.092	515.3	515.6	522.1	-0.059
0.45	0.8377	0.8379	1.074	1.074	1.074	510.8	511.8	505.4	-0.074
0.50	0.8452	0.8454	1.060	1.060	1.060	501.9	502.3	502.0	-0.098
0.60	0.8660	0.8661	1.038	1.038	1.038	464.1	464.7	460.5	-0.179
0.70	0.8989	0.8989	1.021	1.021	1.024	395.8	396.6	400.8	-0.321
0.80	0.9510	0.9510	1.008	1.008	1.008	311.4	312.0	308.0	-0.543
0.90	1.0304	1.0304	1.001	1.001	1.001	240.0	240.5	241.5	-0.829
1.00	1.1362	1.1362	1.000	1.000	1.000	210.1	210.1	210.1	-1.000

$\epsilon_r = 10$, $a/h = b/L$, $L_1 = 0$. M-M denotes the mode-matching method results with $N = 9$. R denotes the Rayleigh-Ritz method results with $N = 100$.

TABLE II
The Computed Values of Normalized QUASI-TE_{0nm}-Mode Frequencies $\omega L/c$ versus the Number of Basis Functions N for the Mode-Matching Method

N	ω/c L					
	TE_{011}	TE_{012}	TE_{021}	TE_{022}	TE_{013}	TE_{031}
1	4.07981	——	——	——	——	——
2	4.32412	5.15487	——	——	——	——
3	4.40270	5.15801	——	7.49754	7.90860	8.19561
4	4.42289	5.16582	7.15565	7.45886	7.91231	8.21483
5	4.42405	5.16837	7.23677	7.46290	7.90435	8.21437
6	4.42393	5.16866	7.27404	7.46977	7.89954	8.21781
7	4.42488	5.16866	7.28842	7.47364	7.89744	8.22207
8	4.42560	5.16869	7.29109	7.47431	7.89711	8.22418
9	4.42575	5.16872	7.29072	7.47413	7.89722	8.22449
R	4.43245	5.17032	7.31309	7.49125	7.94563	8.25259

The DR is situated in a cylindrical cavity. $\epsilon_r = 10$, $a/h = b/L = 1.00$, $h/L = 0.25$. M-M and R have the same meanings as in Table I

Table II gives results of computations of higher order quasi-TE_{0nm}-mode frequencies for the same resonant system and $p = 0.25$. Convergence of the mode-matching method is investigated versus number of basis functions (N). In the last row of Table II results obtained by the Rayleigh–Ritz method are shown. It is seen that the discrepancy between frequency values obtained by these two methods is not greater than 0.6 percent for the first six modes. Results of higher order frequencies and the electric energy filling factor computations versus $p = h/L$ values are shown in Fig. 2. Computations were performed by the Rayleigh–Ritz method. We can observe in Fig. 2 that the frequency curves show the intervals of plateaus alternating with intervals of steep slope. In the intervals of steep slope the p_e values are close to unity and the modes are of the interior type, while in the intervals of plateaus the modes become of the exterior type. This phenomenon has already been reported in [1] and [12].

Results of computations for the resonant system having $\epsilon_r = 35$, $a/h = 1$, $b/L = 1$, and $L_1 = 0$ are presented in Table III. For $p \geq 0.25$ results of computations agree very well (similarly as for $\epsilon_r = 10$). For $p < 0.20$, the differences increase. This is understood, since for small p values and high ϵ_r values the convergence of the classical Rayleigh–Ritz method becomes poor [8]. We can observe that when the DR is placed at the cavity bottom,

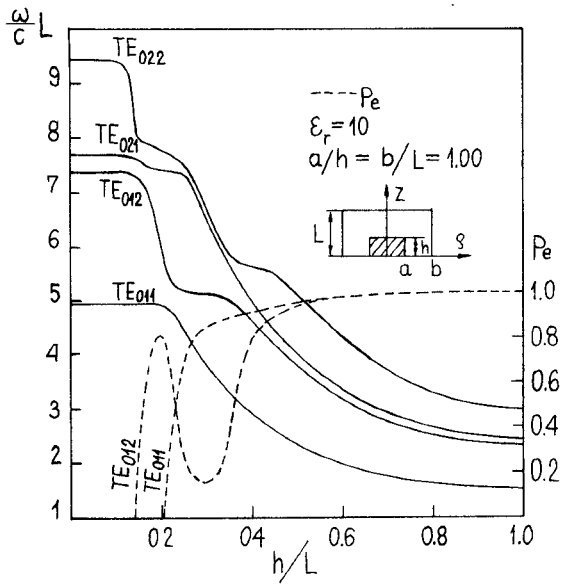

Fig 2. The lowest quasi-TE_{0nm}-mode normalized frequencies ($\omega L/c$) and the electric energy filling factor values p_e versus the h/L ratio for a cylindrical DR situated at the cavity bottom. $\epsilon_r = 10$, $a/h = 1$, $b/L = 1$. The results have been obtained by means of the Rayleigh–Ritz method with $N = 100$.

TABLE III
THE COMPUTED VALUES OF NORMALIZED $TE_{01\delta}$-MODE FREQUENCIES f/f_{rod}, THE ELECTRIC ENERGY RATIO p_e^{-1}, A, AND C_m COEFFICIENTS VERSUS h/L FOR A DR SITUATED IN A CYLINDRICAL CAVITY

h/L	f/frod		Pe⁻¹			A			Cm
	M-M	R	M-M(I)	R(I)	R(D)	M-M(I)	R(I)	R(D)	M-M
0.00	0.0000	0.0000	∞	∞	∞	665.7	665.7	665.7	-1.000
0.10	0.6692	0.6692	659.2	1410.	1413.	661.0	663.1	663.1	-0.998
0.125	0.8271	0.8332	2.754	15.37	15.48	428.2	580.1	586.5	-0.578
0.15	0.8416	0.8525	1.041	1.046	1.047	286.4	265.4	253.5	-0.009
0.20	0.8429	0.8470	1.028	1.029	1.029	287.5	278.2	286.0	-0.006
0.25	0.8440	0.8459	1.024	1.025	1.025	287.2	287.2	297.0	-0.006
0.30	0.8449	0.8459	1.022	1.023	1.023	287.0	288.5	278.9	-0.009
0.40	0.8487	0.8491	1.019	1.019	1.019	286.5	287.0	291.1	-0.026
0.50	0.8567	0.8568	1.013	1.014	1.014	279.6	280.2	279.6	-0.065
0.60	0.8722	0.8723	1.010	1.010	1.010	259.0	259.8	257.1	-0.144
0.70	0.9006	0.9006	1.006	1.006	1.006	219.5	220.8	223.2	-0.288
0.80	0.9487	0.9487	1.003	1.003	1.003	170.4	171.5	169.0	-0.517
0.90	1.0255	1.0255	1.000	1.000	1.000	129.1	129.8	130.2	-0.818
1.00	1.1313	1.1313	1.000	1.000	1.000	112.5	112.5	112.5	-1.000

$\epsilon_r = 35$, $a/h = b/L = 1.00$, $L_1 = 0$. M-M and R have the same meanings as in Table I.

as was the case for the resonant systems considered so far, A-factor values are smaller than that of the empty cavity. If the DR is placed far from the cavity walls, A-factor values can be much higher, as is seen in Fig. 3, which presents results of computations for a DR situated centrally in a cylindrical cavity. The Rayleigh–Ritz method has been used for computations. Maximum A-factor values appear for cavity dimensions 5–6 times greater than the DR ones ($p = 0.15$–0.20). For these p values the $TE_{01\delta}$ mode is interior since the electric energy filling factor values are high ($p_e \geq 0.85$). For p values smaller than 0.15, the $TE_{01\delta}$ mode becomes exterior and A-factor values tend rapidly to the value characteristic of an empty TE_{011} mode cavity.

In Table IV results of computations for open-type nonradiating DR structures are presented. For all cases considered in Table IV, discrepancy between frequency values obtained by the modified Rayleigh–Ritz method and the mode-matching method is not greater than 0.15 percent. A-factor values agree to within

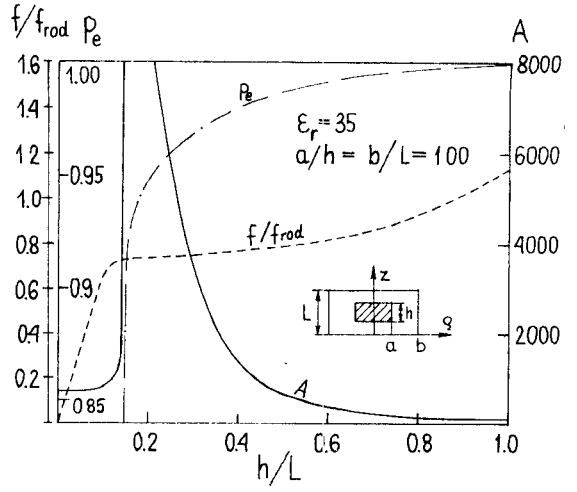

Fig 3. Normalized $TE_{01\delta}$-mode resonant frequency (f/f_{rod}), the electric energy filling factor values p_e, and A-factor values versus the h/L ratio for a cylindrical DR situated centrally in cylindrical cavity. $\epsilon_r = 35$, $a/h = 1$. The results have been obtained by means of the Rayleigh–Ritz method with $N = 100$.

TABLE IV
THE COMPUTED VALUES OF NORMALIZED $TE_{01\delta}$-MODE FREQUENCIES f/f_{rod}, THE ELECTRIC ENERGY RATIO p_e^{-1}, A, AND C_m COEFFICIENTS VERSUS L/λ_{rod} FOR AN OPEN-TYPE DR STRUCTURE

a/h	L/λrod	f/frod		Pe⁻¹			A			Cm
		M-M	R(C)	M-M(I)	R(I)	R(C)	M-M(I)	R(I)	R(D)	M-M
0.25	.2872	1.0000	1.0000	1.032	1.032	1.032	959.9	959.9	959.9	-.178
	.30	0.9927	0.9927	1.033	1.033	1.033	1077.7	1083.3	1071.6	-.156
	.35	0.9773	0.9774	1.042	1.042	1.042	1739.2	1744.7	1754.1	-.059
	.40	0.9747	0.9749	1.051	1.051	1.050	2022.4	2038.5	2058.3	-.033
	.45	0.9685	0.9689	1.061	1.061	1.059	2081.0	2096.6	2140.1	-.024
	.50	0.9660	0.9664	1.084	1.079	1.074	1954.0	2002.2	2137.9	-.030
0.50	.1711	1.0000	1.0000	1.012	1.012	1.012	294.4	294.4	294.4	-.344
	.20	0.9583	0.9583	1.015	1.015	1.016	445.7	447.4	444.7	-.189
	.25	0.9357	0.9357	1.022	1.022	1.022	660.1	661.7	667.4	-.054
	.30	0.9297	0.9298	1.026	1.026	1.027	722.0	723.2	730.5	-.023
	.35	0.9272	0.9276	1.030	1.029	1.029	737.3	739.2	737.2	-.013
	.40	0.9260	0.9264	1.033	1.032	1.032	741.0	743.5	760.3	-.008
	.45	0.9251	0.9257	1.037	1.035	1.034	743.9	743.4	744.3	-.009
	.50	0.9240	0.9251	1.042	1.038	1.036	743.6	741.5	744.3	-.014
1.00	.1177	1.0000	1.0000	1.004	1.004	1.004	119.6	119.6	119.6	-.582
	.15	0.8984	0.8984	1.006	1.006	1.007	189.7	190.3	190.7	-.278
	.20	0.8587	0.8588	1.013	1.013	1.014	262.6	262.8	262.8	-.077
	.25	0.8493	0.8494	1.018	1.018	1.018	280.5	280.7	280.4	-.029
	.30	0.8461	0.8464	1.021	1.020	1.021	284.9	285.0	286.9	-.015
	.35	0.8445	0.8450	1.023	1.022	1.022	286.1	286.1	286.2	-.010
	.40	0.8436	0.8444	1.025	1.024	1.024	285.9	286.6	293.7	-.006
	.45	0.8431	0.8440	1.026	1.025	1.025	285.5	286.3	286.9	-.004
	.50	0.8427	0.8437	1.029	1.025	1.026	286.1	286.7	279.6	-.004
1.50	.1023	1.0000	1.0000	1.002	1.002	1.002	84.0	84.0	84.0	-.720
	.15	0.8314	0.8315	1.007	1.007	1.007	145.0	145.3	145.6	-.243
	.20	0.7968	0.7969	1.013	1.013	1.014	174.2	174.4	175.0	-.082
	.25	0.7871	0.7873	1.018	1.017	1.018	181.4	181.6	183.6	-.035
	.30	0.7834	0.7837	1.021	1.020	1.021	183.4	183.4	183.0	-.019
	.35	0.7817	0.7822	1.023	1.022	1.023	183.8	184.0	188.1	-.011
	.40	0.7808	0.7814	1.025	1.024	1.024	183.9	184.0	183.5	-.007
	.45	0.7802	0.7810	1.027	1.025	1.025	184.3	184.3	180.3	-.007
	.50	0.7795	0.7807	1.029	1.026	1.026	185.0	183.8	184.1	-.010

$\epsilon_r = 35$, $L_1 = 0$, $b = \infty$. M-M denotes the mode-matching method results with $N = 9$. R denotes the modified Rayleigh–Ritz method with $N = 25$.

0.75 percent (except one value for $a/h = 0.25$ and $L/\lambda_{rod} = 0.50$) and p_e factor values agree to within 0.40 percent.

The results of normalized $TE_{01\delta}$-mode frequency computations versus L/λ_{rod} values are presented in Fig. 4 together with experimental data for one DR structure. Experiments have been performed for a DR having $h = 4.00$ mm, $a = 3.42$ mm, and $\epsilon_r = 34.2$. We can note that systematic 0.01 mm error of DR dimensions for the DR used in experiments causes ~ 0.3 percent

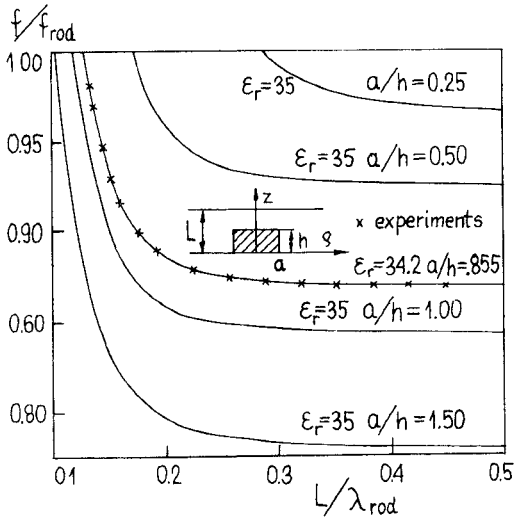

Fig. 4. Normalized $TE_{01\delta}$-mode resonant frequencies (f/f_{rod}) versus the L/λ_{rod} ratio for open-type DR's. $\epsilon_r = 35$, $b = \infty$, $L_1 = 0$. The results have been obtained by means of the modified Rayleigh–Ritz method with $N = 25$.

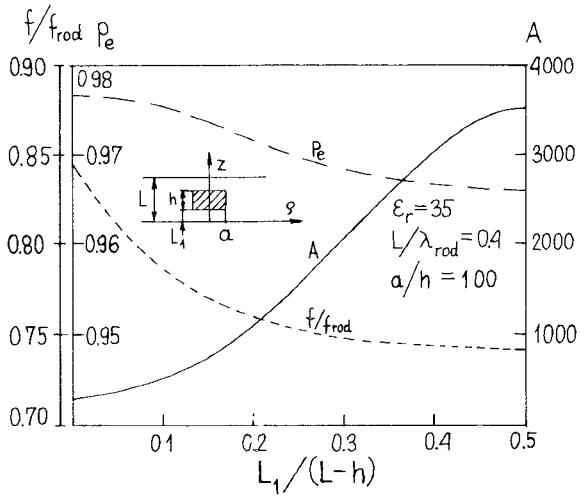

Fig. 5. Normalized $TE_{01\delta}$-mode resonant frequency (f/f_{rod}), the electric energy filling factor values p_e, and A-factor values versus the distance $L_1/(L-h)$ between the dielectric resonator and the metal plate. $\epsilon_r = 35$, $L/\lambda_{rod} = 0.4$, $a/h = 1$. The results have been obtained by means of the modified Rayleigh–Ritz method with $N = 25$.

error in frequency value. Such error is greater than errors of computations.

In Fig. 5 we present the influence of the distance L_1 between the dielectric resonator and the metal plate on the $TE_{01\delta}$-mode frequency, p_e, and A factor values. The results have been obtained by the modified Rayleigh–Ritz method. As with cavity-type resonators, the A coefficient reaches a maximum for symmetric DR position. For the resonant system shown in Fig. 5, the maximum value of A is approximately equal to 3500. The value $L/\lambda_{rod} = 0.4$ corresponds to the value of $h/L = 0.294$. For a centrally filled cavity with $h/L = 0.3$ (Fig. 3) the value of A is only 3 percent less than for an open-type structure, so in this case conductor losses in the lateral surface of the cavity constitute only a small part of total conductor losses. The influence of the lateral surface on the resonant frequency, for the case considered here, is smaller than 0.5 percent. We find from the results shown in Table IV ($L/\lambda_{rod} = 0.4$ and $a/h = 1$) that for a similar case and $L_1 = 0$ the influence of the lateral surface on A-factor values

TABLE V
THE COMPUTED VALUES OF NORMALIZED $TE_{01\delta}$-MODE FREQUENCIES f/f_{rod}, THE ELECTRIC ENERGY RATIO p_e^{-1}, A, AND THE TEMPERATURE COEFFICIENTS FOR MIC DIELECTRIC RESONATOR STRUCTURES

L_1/h		ϵ_{rs}	f/f_{rod}	P_e^{-1}	A	C_d	C_m	C_{L_1}	τ_{ϵ_r}	$\tau_{\epsilon_{rs}}$
0.06	$L=L_1+h$	1.0	.9677	1.004	130.0	-.4384	-.5637	+.0021	-.4978	-.00002
		2.5	.9677	1.004	130.0	-.4384	-.5636	+.0021	-.4978	-.00005
		10.0	.9676	1.004	130.0	-.4384	-.5632	+.0046	-.4976	-.00019
	$L=4\lambda_{rod}$	1.0	.8251	1.024	316.4	-.9742	-.0052	-.0206	-.4883	-.00001
		2.5	.8251	1.024	316.3	-.9742	-.0052	-.0206	-.4883	-.00002
		10.0	8250	1.024	315.5	-.9740	-.0052	-.0208	-.4883	-.00010
0.24	$L=L_1+h$	1.0	.9057	1.007	180.5	-.4920	-.5556	+.0477	-.4965	-.00055
		2.5	.9049	1.009	179.3	-.4920	-.5536	+.0456	-.4958	-.00140
		10.0	.9010	1.018	173.1	-.4915	-.5431	+.0346	-.4911	-.00603
	$L=4\lambda_{rod}$	1.0	.7851	1.025	525.6	-.9448	-.0075	-.0477	-.4878	-.00033
		2.5	.7847	1.026	517.0	-.9437	-.0075	-.0488	-.4873	-.00085
		10.0	.7827	1.032	476.0	-.9379	-.0076	-.0545	-.4845	-.00357

$\epsilon_r = 35$, $a/h = 1.00$, $b = \infty$. The results have been obtained by means of the modified Rayleigh–Ritz method with $N = 25$.

is smaller than 0.4 percent and smaller than 0.15 percent of frequency values.

As the next example, we consider open-type DR's on a dielectric substrate. The results of computations using the modified Rayleigh–Ritz method are shown in Table V. We note that for small thickness of the substrate ($L_1 = 0.06h$) its permittivity has a negligible influence on the resonant $TE_{01\delta}$-mode frequency of the system. Even for relatively thick substrates ($L_1 = 0.24h$) the influence of ϵ_{rs} on the frequency value is not greater than 0.5 percent for the DR structures considered in Table V. Now we consider the problem of thermal stabilization of the resonant frequency for different DR systems. Using the numerical results presented in Tables I–V and equations (8)–(10) we can determine all the coefficients required for the calculation, assuming that material properties are known. It is seen from the results presented in Tables III and IV that for $\epsilon_r = 35$ and $a/h \geq 0.5$ the C_m coefficient values are smaller than 0.01 if the distance L and radius b are sufficiently large. Assuming that $\alpha_m \leq 25$ ppm/°C (as occurs for all metals used in practice), the influence of thermal expansion of the metal shield on τ_f is smaller than 0.25 ppm/°C in such cases. Such systems are useful for measurements of τ_{ϵ_r} coefficients if the α_d coefficient is known. For a cavity almost completely filled with a dielectric, the values of C_m and C_d are $C_m \approx -1$ and $C_d \approx 0$. Theoretically such systems can also be useful for measurements if the α_m value is known. Since the C_m and C_d coefficients change their values with the distance L, it is difficult in practice to get thermally stable tunable DR systems. Theoretically it is possible to get $\tau_f = 0$ for tunable systems if $L_1/h \approx 0$, $\alpha_m = \alpha_d$, and $\tau_{\epsilon_r} = -2\alpha_d$, assuming that the mode of interest is interior ($p_e \approx 1$).

As the last example we consider $TE_{01\delta}$-mode resonant systems whose substrate is made of material having any value of the imaginary part of permittivity $\mathrm{Im}(\epsilon_{rs})$. Complex frequency mode-matching method has been used for computations [4]. The results of computations are shown in Fig. 6. Maximum value of $\mathrm{Im}(\epsilon_{rs}) \approx 10^8$ corresponds approximately to silver conductivity. We can notice that for $\mathrm{Im}(\epsilon_{rs}) > 10^4$ the substrate can be considered as made of metal (more precisely, the substrate can be treated as made of metal if its thickness is many times greater than the depth of penetration for particular $\mathrm{Im}(\epsilon_{rs})$ and frequency values). For such values of $\mathrm{Im}(\epsilon_{rs})$ one can use the approximate methods of the resonant frequency and Q_u-factor computations (the same as described for low-loss systems). The approximate methods can also be used for computations if $\mathrm{Im}(\epsilon_{rs})$ values are

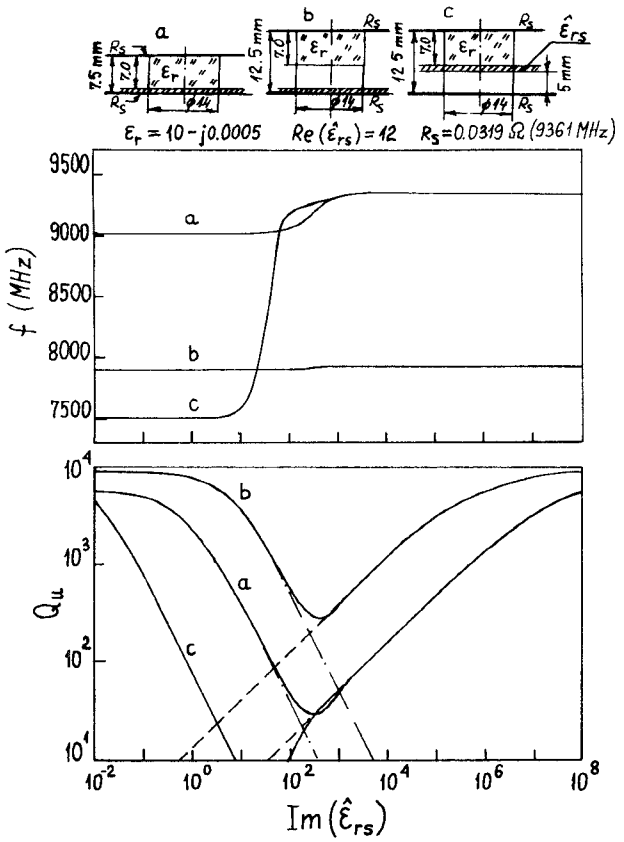

Fig. 6. $TE_{01\delta}$-mode frequencies and unloaded Q_u-factor values versus the imaginary part of substrate permittivity $\mathrm{Im}(\epsilon_{rs})$. The results have been obtained by means of the complex mode-matching method with $N = 5$. Dashed lines indicate Q_u-factor values computed by means of perturbation theory with real mode-matching method.

small. For the systems considered in Fig. 6, a small $\mathrm{Im}(\epsilon_{rs})$ value means any value smaller than 1. As is seen, only in relatively narrow ranges of $\mathrm{Im}(\epsilon_{rs})$ must complex frequency formalism be used for the resonant frequency and Q_u-factor computations.

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Composite Inductive Posts in Waveguide—A Multifilament Analysis

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***Abstract*—A multifilament moment solution for the analysis of composite dielectric posts in rectangular waveguide is presented. This method permits the analysis of inductive posts composed of disparate regions, each with its own homogeneous complex permittivity. The solution uses the fields generated by sets of fixed-amplitude current filaments to simulate both the field scattered by the posts and the field inside every homogeneous region comprising the posts. Point matching the electric and magnetic fields on the boundaries between regions of different permittivity yields the as yet unknown amplitudes for the current filaments. These currents can in turn be used to calculate field-related parameters of interest such as the scattering matrix and the equivalent circuit parameters. Inductive posts of any shape, composition, size, location, and number can be handled by this method accurately and with very good numerical efficiency. The results obtained are in good agreement with the few cases for which data are available. They also behave well in the limiting cases studied. The solution is further applied to other situations for which no experimental or calculated results are known.**

I. Introduction

The study of dielectric waveguide posts of the inductive type is gaining momentum, with a number of works published. In a recent work by the authors [1], a rapidly converging moment solution for the analysis of homogeneous dielectric posts of the inductive type in rectangular waveguide has been suggested. The solution in [1] is numerically efficient and general in that inductive posts of arbitrary smooth shape, size, location, and number can be handled. It is, however, restricted to homogeneous posts and, furthermore, the formulation introduced there deals explicitly with a single post.

A list of useful references to a large body of work on homogeneous dielectric posts can be found in [1]. Composite posts, that is, posts homogeneous only in parts, have received much less attention. Perhaps the only analysis of anything that can be classified as a composite post problem is the work initiated by Nielsen [2] and pursued by Gotsis, Vafiadis, and Sahalos [3]. In these works, a circular post composed of two concentric regions, each with its own permittivity, was analyzed with the goal of

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